Star Pattern Identification Aboard an Inertially Stabilized Spacecraft

Jean Claude Kosik*
Centre National d'Etudes Spatiales, Toulouse, 31055 France

A number of algorithms are presented for the identification of stars for the joint French-Soviet Gamma and Granat spacecraft. Comparative statistical analyses are presented. One algorithm, employing vectorial searches, has been found to be much more efficient than other candidate algorithms examined.

Introduction

PRECISE attitude determination of spacecraft is achieved through the use of star sensors. The use of star-sensor data requires the identification of stars from observed patterns by the sensor, which are then compared with known patterns obtained from a star catalog that contains the position, magnitude, spectral classification, and other data, often for stars of magnitude down to 7 or 9. A study of the efficacy of various star identification algorithms is the subject of this paper.

The present studies were carried in support of the joint French-Soviet Gamma and Granat spacecraft. The Gamma spacecraft carries a rastering star sensor with an image dissector tube that operates in search mode only and records the position and magnitude of star positions and magnitudes downlinked in a rastering cycle; only four stars are available for each cycle. This Gamma star sensor is able to detect stars down to magnitude 9.

The star sensor on board the Granat spacecraft is capable of detecting simultaneously the positions and magnitudes of as many as 30 stars of magnitude 6 or brighter.

Most of the algorithms for star identification in the literature[1-4] were developed for the identification of measured stars by slit star sensors often used onboard spinning spacecraft, for which the attitude accuracy requirements are often quite modest. In this work, we describe candidate algorithms developed for the aforementioned projects that were tested using observations taken with Earth-bound star sensors. One of these algorithms, the polygon-match technique, is discussed briefly by Gottlieb.[5]

This technique is also used by Junkins et al.[6] for pairs of stars, each observed from a different sensor. The other algorithms have been developed especially for this study and use angular and vectorial matching techniques. In the particular case of the Gamma spacecraft, the magnitudes of the stars were included in the algorithms for unambiguous identification in the Milky-Way regions.

The first part of this paper presents the candidate algorithms on an intuitive basis. In the second part, a statistical analysis of these algorithms is carried out to examine their efficiencies.

Candidate Star-Identification Algorithms

Polygon Match

This technique is discussed briefly by Gottlieb.[5] From a set of measured stars, we arbitrarily select two stars, 1 and 2, and compute the corresponding angular separation d_m^{12}, defined as

$$d_m^{12} = \cos^{-1}(S_1 \cdot S_2)$$

where S_1 and S_2 are the directions of the two stars as measured by the star sensor. We then look for all pairs of stars (i,j) in a finite region of the catalog around the approximate boresight of the star sensor, such that

$$|d(i,j) - d_m^{12}| \le \epsilon$$

where ϵ is the uncertainty in the distance measurements of the star sensors and $d(i,j)$ is the angular separation calculated for entries i and j in the star catalog. The number of pairs from the catalog that satisfy this condition increases from the galactic pole to the galactic plane. When only one pair is found and no other information is available, there are two possible identifications of the stars. If more than two stars are observed by the star sensor, we select a third star, 3, from which we compute two more separations, d_m^{13} and d_m^{23}. We now look for a third star in the catalog that can be combined with the previous pair such that

$$|d(i,j) - d_m^{13}| \le \epsilon \text{ and } |d(j,k) - d_m^{23}| \le \epsilon$$

or

$$|d(j,k) - d_m^{13}| \le \epsilon \text{ and } |d(i,k) - d_m^{23}| \le \epsilon$$

The preceding conditions eliminate the ambiguity encountered with two stars unless we have

$$d_m^{23} = d_m^{13}$$

In this case, or in the case where more than one solution is found, it is necessary to select a fourth or perhaps even a fifth star and repeat this process until an unambiguous identification is achieved. The time needed to perform these calculations increases with the number of measured stars and with the number of catalog stars in the boresight region. This step-by-step process, looking first for pairs, then for triplets, and so forth, is much less time consuming than trying to match n stars simultaneously.

Pole Technique

Suppose we have a finite number of measured stars, say five stars, for example. We choose a given star (labeled 1 in Fig. 1) and call it the pole star. The remaining measured stars we call satellite stars. To this pole star and its satellites we associate the distances d_m^{1n}, $n = 2,\ldots, 5$. For each measured separation, we associate the set ϵ^{1n} of pairs (i,j) in the catalog that satisfy the criterion

$$|d(i,j) - d_m^n| \le \epsilon$$

For distances d_m^{12}, d_m^{13}, d_m^{14}, d_m^{15}, we have the sets $\mathcal{E}^{12}$, $\mathcal{E}^{13}$, $\mathcal{E}^{14}$, and $\mathcal{E}^{15}$. If a star from the catalog is a possible candidate for the measured pole star 1, it should be a member of at least one pair in each of the sets $\mathcal{E}^{1n}$, $n = 2,\ldots, 5$. Thus, it should belong

Received Jan. 25, 1988; revision received Dec. 9, 1988.

*Engineer, Mathematics Division.

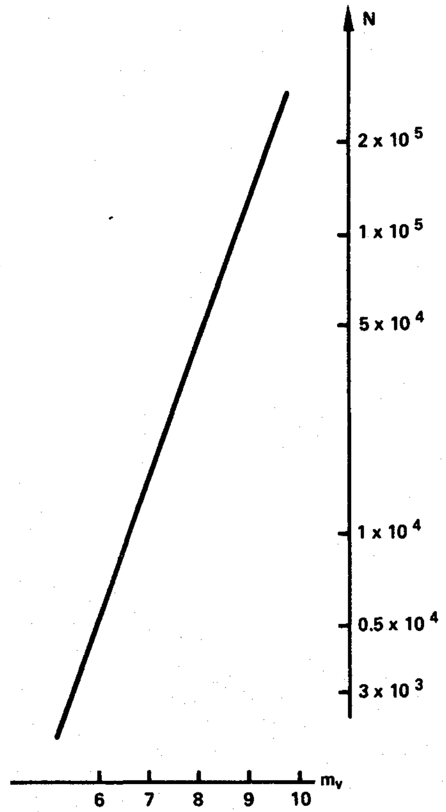

Fig. 1 For a given number of measured stars we select a star that is called the pole star and associate to this star the distances to the other stars. For each distance there is a set of catalog pairs, and the candidate star of the catalog as a pole star belongs to the intersection of the sets.

to the intersection of these sets $\cap_{n=2}^{n=5} \mathcal{E}^{1n}$. If this intersection is a singleton, i.e., if it contains only a single element, then the identification is unambiguous. If there is more than one star in this intersection, then it may be necessary to increase the number of satellite stars until a unique identification is achieved. If a successful match is achieved, we proceed with stars 2–5 in the same way.

It has been our experience that this algorithm performs well if the number of observed stars in the field of view is greater than seven. When this is not the case, a possible improvement in the likelihood of a unique identification is obtained as follows.

Each of the candidate pole stars belongs to a pair in each of the sets $\mathcal{E}^{1n}$, $n = 2,\ldots, 5$. In each of these $\mathcal{E}^{1n}$, we call the other member of the pair associated with a candidate pole star the associated star. We define $\mathcal{E}'^{1n}$ to be the set of all stars in $\mathcal{E}^{1n}$ associated with any of the candidate pole stars. In general, the number of stars in $\mathcal{E}'^{1n}$ will be much less than the number of stars in $\mathcal{E}^{1n}$. We now select star 2 as our new pole star and begin searching for a candidate pole star 2 and satellite stars 1, 3, 4, and 5, but only searching over the much smaller sets $\mathcal{E}'^{1n}$ (and, effectively, the set of candidate stars for 1). The intersection of successful separation matches is determined again, and if this is a singleton, then 2 will have been correctly identified, and the other stars then identified in turn.

Polygon Angular Match

We can improve the polygon technique if we have a crude estimate of the attitude. With this information, we calculate the positions of the catalog stars that belong to the region of the boresight in the approximate frame of reference of the star sensor (the local catalog). For each pair of observed stars, we look for all pairs of the local catalog that satisfy the corresponding distance criterion. For each pair, observed or catalog, we define a vector v, which is the difference in the focal-plane coordinates of the two stars in the pair. Acceptable pairs of catalog vectors should satisfy the condition

$$V_c\,(i,j) \cdot V_m \geq 1 - \mu$$

when $V_c\,(i,j)$ and V_m are the catalog and observed vectors, respectively, and (i,j) labels the catalog pair. The uncertainty in the attitude is represented by μ. For two stars in the catalog that satisfy the separation criterion, S_i and S_j, we have two possible vectors

$$V_c\,(i,j) \equiv S_i - S_j \text{ and } V_c(j,i) \equiv S_j - S_i$$

But if the vectors are parallel to the measured vector V_m, then if in one case we get

$$V_c\,(i,j) \cdot V_m \geq 1 - \mu$$

in the other we will obtain

$$V_c\,(j,i) \cdot V_m \geq -1 + \mu$$

There is now a one-to-one correspondence between the catalog stars and the measured stars, and the ambiguity disappears.

If more than one pair is found, it is necessary to add more information, introducing a new measured star. We proceed in the following way. For measured stars 1 and 2, we look for acceptable stars as before, then we choose stars 1 and 3 and look for the other vectors. We now compare the $V_c^{ki} \cdot V_c^{kj}$ with the $V_m^{12} \cdot V_m^{13}$, where k denotes the candidate star for 1, and so forth. If we can find acceptable catalog stars, then the scalar products should be equal within the uncertainty of the measurements, which depends on the accuracy of the star sensor. If there still remains some ambiguity, we add another star and combine V_m^{12} with V_m^{13}, then V_m^{13} with V_m^{14}, and finally, V_m^{12} V_m^{14}. The correct catalog stars should give the correct scalar products. Usually it is not necessary to use four measured stars. Three will usually suffice as this algorithm is quite powerful. Moreover, the use of vectors does not require additional calculations as these are already performed in order to select the stars that can be observed by the star sensor.

Orientation-Angle-Magnitude Algorithm

In the previous algorithms, the magnitude of the stars was not taken into account. It is possible to introduce the magnitude and define a "magnitude distance" between the observed stars and the catalog stars. We then search for matches between pairs of observed and catalog stars as in the polygon-angular match algorithm but with the difference vectors augmented to have an additional component, which is the difference in the star magnitude. To get adequate results, it is necessary to compensate the instrumental magnitude from the star sensors in order that a meaningful comparison of "magnitude distances" can be made. Since the polygon-angular match algorithm is already very powerful, the additional criterion permits the efficient use of very large star catalogs (down to magnitude 9) even with poor information (only four stars and their magnitudes). At this magnitude there can be as many as 1000 stars in the neighborhood of the boresight, and the computational burden increases dramatically. If we have even some very crude information on the attitude, it is possible to sort the catalog and keep only those stars that are in the neighborhood of the measured stars, thereby reducing the computational burden dramatically.

Statistical Analysis

The ease of identification of a star pattern is directly related to the number of observed stars available as well as the number of stars in the catalog. The number of stars in the field of view increases with the magnitude limit of the detector, and only a small sample of this population will be registered by the detector. Figure 2 gives the total number of stars in the star catalog as a function of the visual magnitude limit, from which we can deduce the average number of stars per square degree. The real number of stars depends on the location of the detector in the sky, whether at the galactic equator or at the galactic pole. To get some insight into the efficiency of the algorithms, we have plotted several distributions for a 6 × 6 deg field of view using the Bright Star Catalog.

In Figs. 3 and 4, we have plotted the number of pairs as a function of the separation in a 6 × 6 deg field of view for two different regions of the sky, near the galactic equator and away from the galactic equator. The possible range in separation is from 0 to 10 deg. There is a variation of a factor 7 in the density of pairs over the celestial sphere. In Figs. 5 and 6, we have plotted for the same parameters, respectively, the number of pairs as a function of the orientation. Here there is

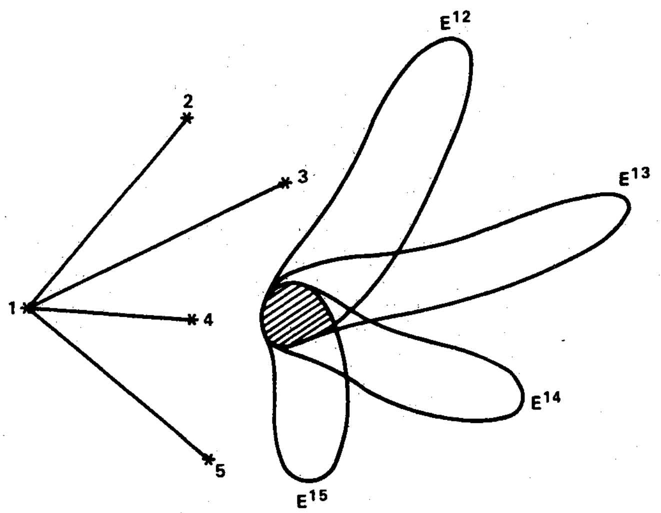

Fig. 2 The logarithmic scale gives the number of stars in the sky down to a given visual magnitude.

a factor of 10 between the galactic equator and the galactic polar region. For the separations, a step size of 0.1 deg was chosen and for the orientation, a step size of 5 deg. This means that if the star sensor has an accuracy of 1 arc min in the measured separation, one can expect $n/6$ pairs that would satisfy the separation criterion, where n is the number of pairs per separation step size. If we know the crude attitude with an uncertainty of 2°5, the corresponding numbers of Figs. 5 and 6 should be divided by 2.

Using this data base, the following statistical conclusions may be reached for each of the four algorithms already discussed.

Polygon Match

Let us consider the case of a star pattern of N observed stars. We select two stars. As shown by Fig. 3, one can expect approximately 15 pairs within a separation error (ϵ) of 2 arc min in a field of view containing 80 stars. The probability of getting the correct catalog pair is thus 1/15, and to get two correct catalog pairs is 1/225. It is necessary, therefore, to add more information. We now consider three observed stars labeled 1, 2, and 3 with separations d_m^{12}, d_m^{13}, and d_m^{23}. From Fig. 3, we can associate N_1, N_2 and N_3 pairs in the catalog that would meet the separation criterion. We call, $\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_3$ the sets of pairs associated with N_1, N_2, and N_3, respectively. We select arbitrarily a pair of stars (a,b) in set $\mathcal{E}_1$. A possible solution to the match can be obtained, say, with a pair $(a,c) \in \mathcal{E}_2$ and pair $(b,c) \in \mathcal{E}_3$. We now calculate the probability of finding such a triplet. The probability that star $a \in \mathcal{E}_2$ is $2N_2/N$ and likewise the probability that star $b \in \mathcal{E}_3$ is $2N_3/N$. The probability of finding the ordered triplet is given by

$$P(a \in \mathcal{E}_2 \cap b \in \mathcal{E}_3) = 4N_2\, N_3/N^2$$

The probability of finding the triplet with the opposite order is

$$P\,(a \in \mathcal{E}_3 \cap b \in \mathcal{E}_2) = 4N_2N_3/N^2$$

and, finally, the probability of finding the triplet irrespective of order is

$$P\left(\{a \in \mathcal{E}_2 \cap b \in \mathcal{E}_3\} U \{a \in \mathcal{E}_2 \cap b \in \mathcal{E}_3\}\right) = 8N_2N_3/N^2$$

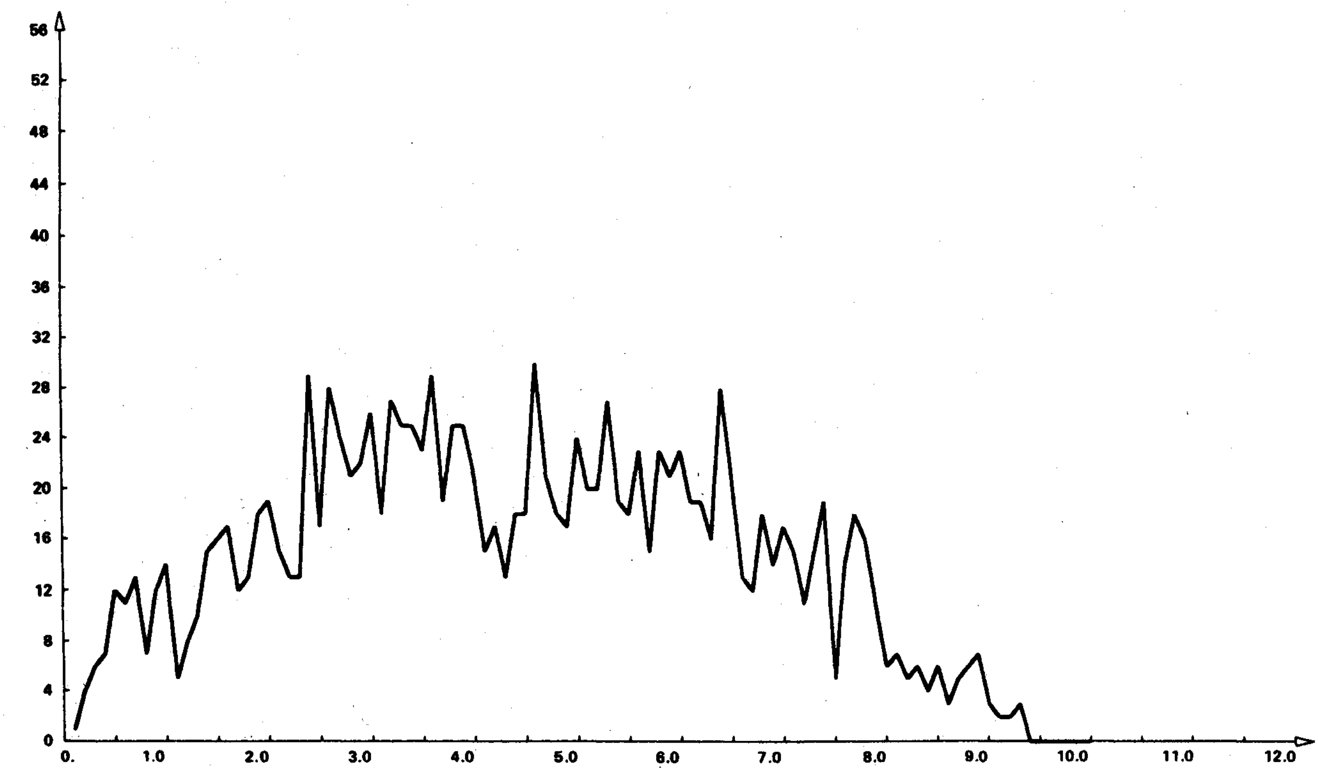

Fig. 3 Number of pairs in a field of view of 6×6 deg as a function of the separation in the galactic plane. A separation interval of 0.1 deg is used, and the separation range of 0–10 deg corresponds to the diagonal of the field of view.

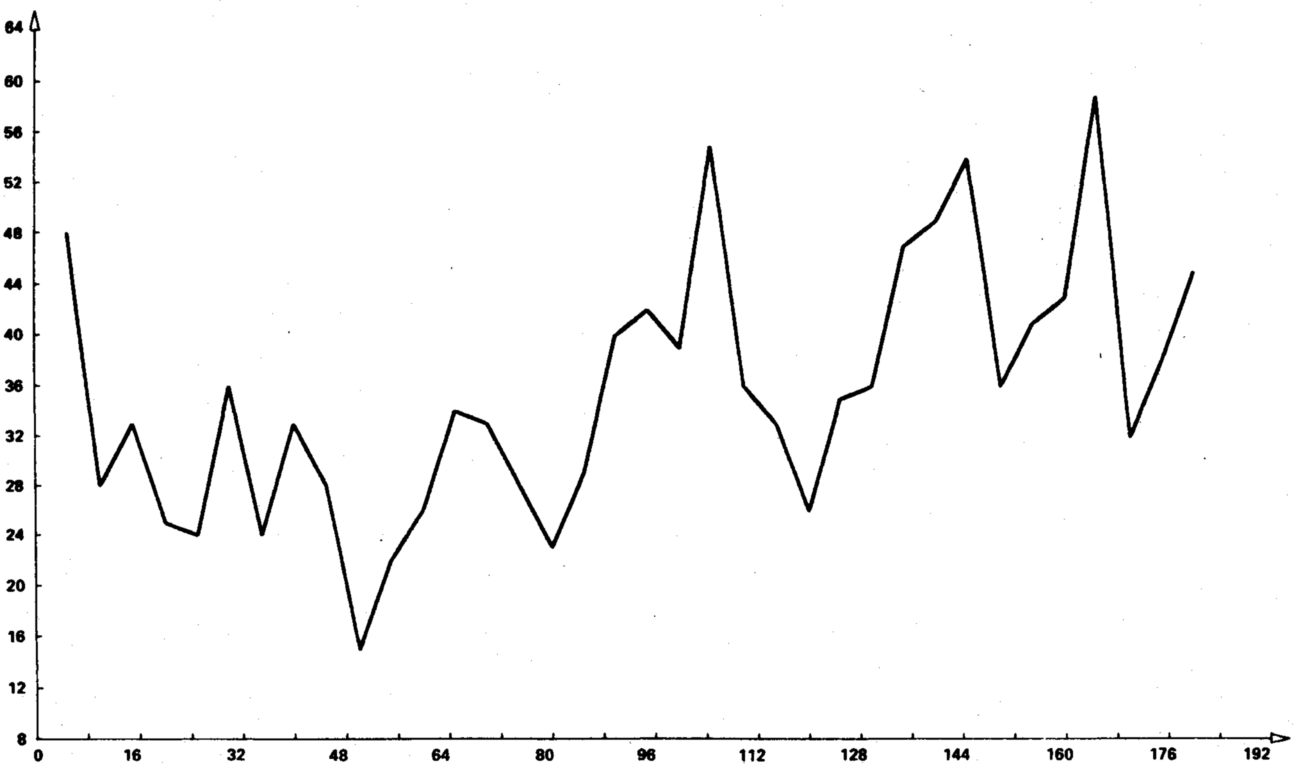

Fig. 4 Number of pairs in a field of view of 6 × 6 deg as a function of the separation outside the galactic plane.

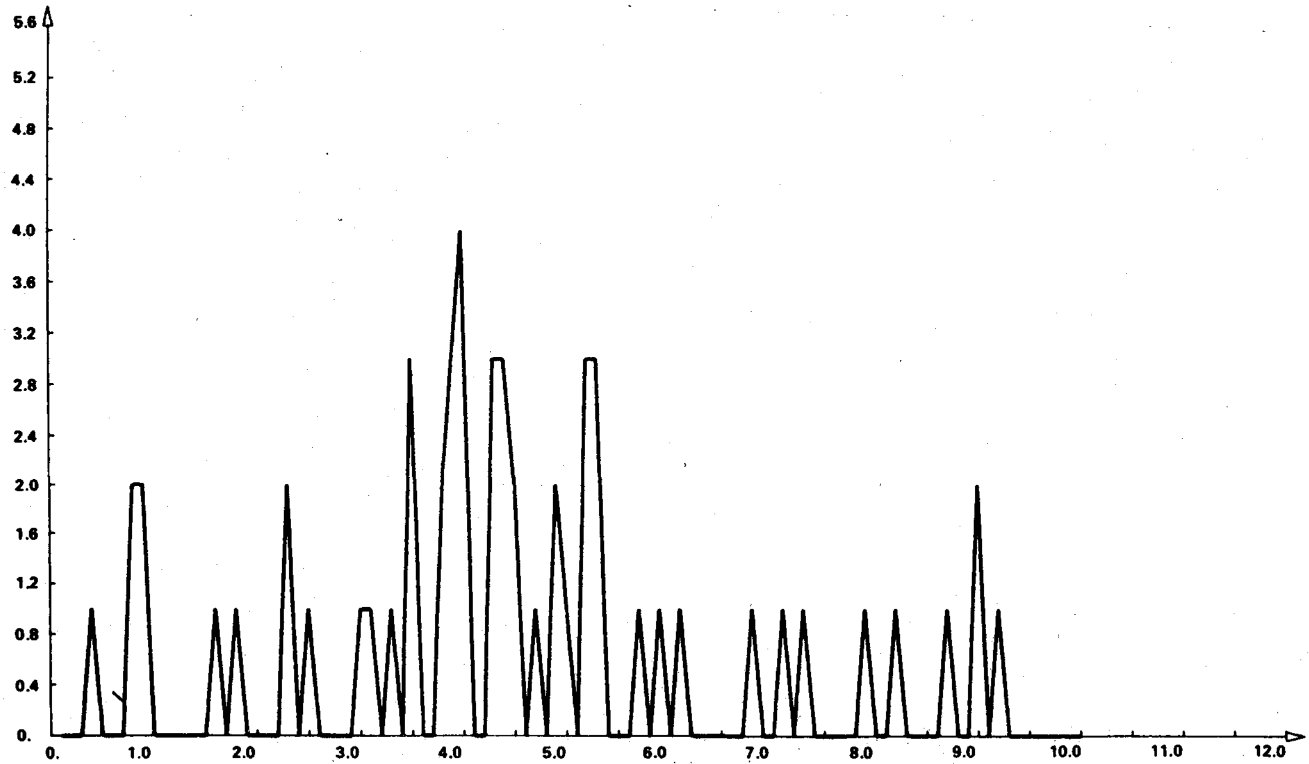

Fig. 5 Number of pairs in a field of view of 6×6 deg as a function of orientation in the galactic plane; a step of 5 deg was used to sample all the vectors.

But star c, which is associated, for example, with star b, and which forms a pair in $\mathcal{E}_3$ must also be found in $\mathcal{E}_2$ and associated with a star as a pair. The corresponding probability for this simultaneous occurrence can be written

$$P\,(c \in \mathcal{E}_2 \cap \{(a,c) \text{ has been found}\}) = 2N_2/N \times 1/N_2 = 1/N$$

For pair (a,b), the probability of finding a triplet given a is thus

$$8N_2N_3/N^2 \times 2/N = 16N_2N_3/N^3$$

There are N_1 couples like (a,b) in set $\mathcal{E}_1$. The probability of

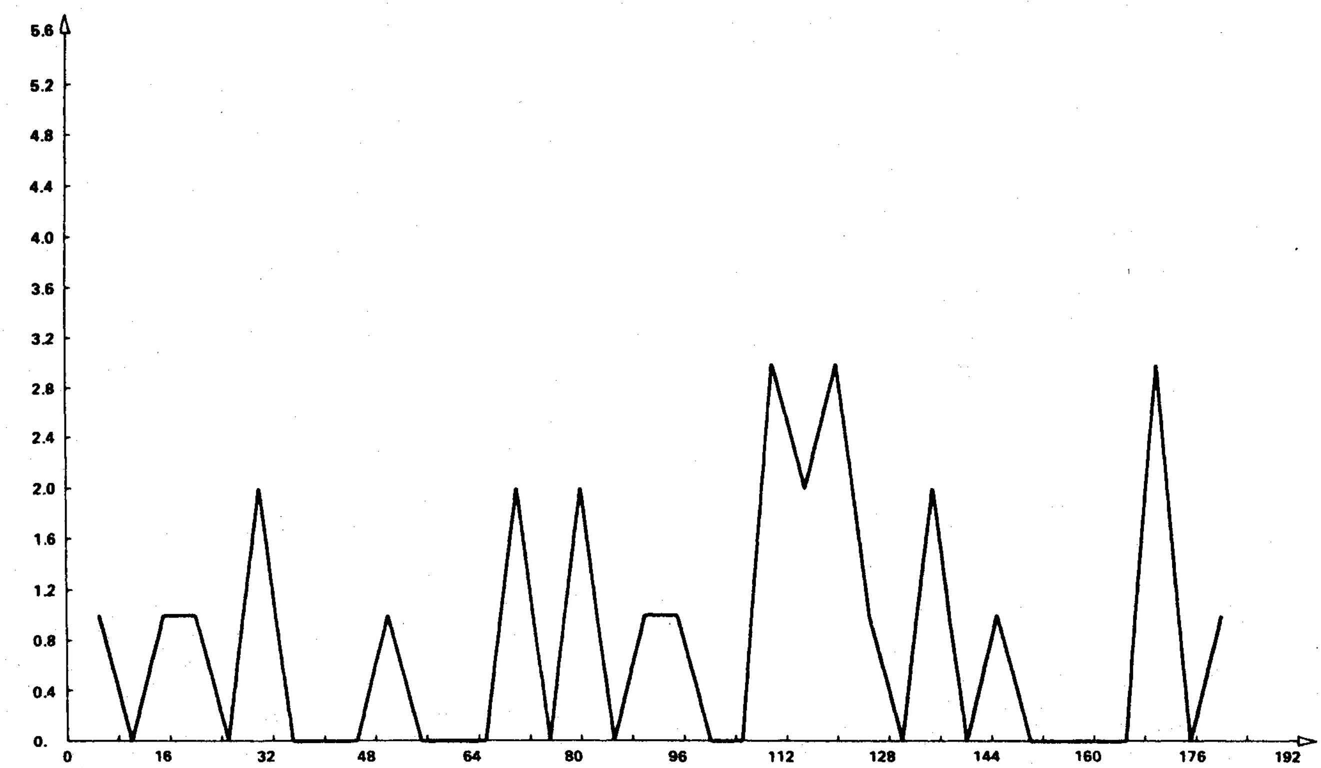

Fig. 6 Similar to Fig. 5 but outside the galactic plane.

finding a triplet is thus

$$16N_1N_2N_3/N^3$$

If one more star is observed, labeled 4, we can take into account the additional separations d_m^{14}, d_m^{24}, and d_m^{34}. We can associate N_4, N_5, and N_6 pairs in the catalog that would meet the separation criterion. We call $\mathcal{E}_4$, $\mathcal{E}_5$, and $\mathcal{E}_6$ the sets of pairs associated with N_4, N_5, and N_6, respectively. A possible solution b for the match can be obtained with a pair $(a,d) \in \mathcal{E}_4$, a pair $(b,d) \in \mathcal{E}_5$, and a pair $(c,d) \in \mathcal{E}_6$. Stars a, b, c belong to sets $\mathcal{E}_4$, $\mathcal{E}_5$, and $\mathcal{E}_6$ respectively. The probability of finding this triplet is

$$P(a \in \mathcal{E}_4 \cap b \in \mathcal{E}_5 \cap c \in \mathcal{E}_6) = 8N_4N_5N_6/N^3$$

the star d must belong to the sets $\mathcal{E}_4$, $\mathcal{E}_5$, and $\mathcal{E}_6$. We now consider the pair $(a,d) \in \mathcal{E}_4$. A possible solution is obtained if d belongs to sets $\mathcal{E}_5$ and $\mathcal{E}_6$ and forms the pairs (b,d) and (c,d). The corresponding probability is

$$P\,(\{d \in \mathcal{E}_3 \cap (b,d) \text{ has been found}\} \cap$$

$$\{d \in \mathcal{E}_6 \cap (c,d) \text{ has been found}\}) = 4/N^2$$

The probability of finding a quartet (a,b,c,d) with $a \in \mathcal{E}_4$, $b \in \mathcal{E}_5$, and $c \in \mathcal{E}_6$ is

$$16N_1N_2N_3/N^3 \times 8N_4N_5N_6/N^3 \times 4/N^2$$

But there are 3! possible combinations with a,b,c and $\mathcal{E}_4$, $\mathcal{E}_5$, $\mathcal{E}_6$. The probability of finding a quartet is

$$2048N_1N_2N_3N_4N_5N_6/N^8$$

Now we assume that we have approximately $N_i = 8$ and $N = 80$. The probability to get a triplet or a quartet is 0.016 or $0.32\ 10^{-6}$, respectively. These results seem to contradict the facts: The probability to obtain a triplet or a quartet is small, but we are sure that the stars we measure are in the catalog unless there is a failure of the star sensor. These results do give a measure of a possible misidentification; the probability of getting two triplets or two quartets is 0.000256 or 10^{-13} respectively. These values show that the algorithm is much more reliable with four stars than with three stars.

Polygon Angular Match

From Fig. 3 we obtain the number N_1 of catalog pairs for a given separation and an uncertainty ϵ in the measurement. Likewise, from Fig. 5, we obtain the number N_2 of catalog pairs for a given orientation. The probability that a couple has the right orientation is

$$2N_2/N(N-1)$$

where $N(N-1)/2$ is the total number of pairs obtained with N stars. The probability of finding a pair with the adequate separation and orientation is

$$2N_1N_2/N(N-1)$$

If we assume a uniform distribution in orientation and take an interval of 2 deg we obtain $2N_2/N(N-1) = 1/180$. With $N_1 = 8$, we obtain an approximate value of 0.05 for the probability.

If we consider three measured stars, they form three vectors, but only two vectors have to be considered, as the third one is deduced from them. The probability of finding the right oriented triplet is the probability of finding the right triplet $16N_1N_2N_3/N^3$ multiplied by the probability of obtaining two right orientations $4N_1\ N_2/N^2(N-1)^2$. The probability is

$$64N_1^2N_2^2N_3/N^5(N-1)^2$$

We previously evaluated the first term to 0.016 and the second term to $(1/180)^2$. The probability of finding a correctly oriented triplet is 0.5×10^{-6}. With these figures, the probability of obtaining two such triplets or of making a misidentification is less than 10^{-13}. These results indicate that this algorithm is very efficient, especially in the case of three stars, and it is our favorite.

Pole Technique

From Fig. 3, we obtain N_1 pairs of catalog stars for separation d_m^{12}, N_2 pairs for separation d_m^{13},..., N_i pairs for separation d_m^{1i}. The corresponding sets $\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_i$ have, respectively, $N_1, N_2, \ldots, N_i$ elements. If we choose a given star of $\mathcal{E}_1$, the probability of finding this star in $\mathcal{E}_2$ is $2N_2/N$ where N is the total number of catalog stars near the boresight. The probability of finding this star in $\mathcal{E}_2, \ldots, \mathcal{E}_i$ is

$$(2N_2/N) \times \ldots \times (2N_i/N)$$

If there are $2N_1$ stars in ϵ_1, the probability of finding a star which belongs to all the sets is

$$2N_1 \bigcap_{i=1}^{i=n} (N_i/N)$$

where n is the number of sets $\mathcal{E}_i$.

If we take $N = 80$ and $N_i = 8$, we obtain a probability of 0.025 for five measured stars. The probability of obtaining more than one solution is 0.0006. These values show that this algorithm is less efficient than the polygon match or the polygon-angular match. To obtain a reliable result, it is necessary to use at least seven stars. In this case the corresponding figures drop down to 0.001 and 0.4×10^{-6}.

Conclusion

For the two projects Gamma and Sigma we first developed the pole algorithm, then the polygon match, and the polygon-angular match. For the Gamma star sensor, the orientation-angle-magnitude algorithm was developed for achieving identification with only four measured stars among a field of catalog stars down to magnitude 9. The pole technique revealed itself as the most complicated code to develop and the least efficient. Without any a priori information on attitude, the polygon match algorithm was simpler and more efficient. However, with some crude information on attitude, the polygon-angular match algorithm gave the best results: its code was nearly as simple as the polygon match code, and the efficiency was greatly improved, as confirmed by our probabilistic approach.

References

[1]Walsh, T. M., and Kenimer, R. L., "Analysis of a Star Field Mapping Technique for Use in Determining the Attitude of a Spin-Stabilized Spacecraft," NASA Langley Research Center, Hampton, VA, NASA TN-D-4637, 1964.

[2]Walsh, T. M., and Keating, J. C., "Attitude Determination of the Spin-Stabilized Scanner Spacecraft," NASA Langley Research Center, Hampton, VA, NASA TN-D-4740, 1967.

[3]Snyder, J. L., and Meyers, G. F., "SAS Attitude Support System," Space Simulation Conference, Paper-79, NASA NAS5-11790, 1979, pp. 909–931.

[4]Wong, E. C., and Lai, J. Y., "Attitude Determination of Galileo Spacecraft from Star Data," *Proceedings of the AIAA Guidance and Control Conference*, AIAA, New York, 1980, pp. 111–124.

[5]Gottlieb, D. M., "Star Identification Techniques," *Spacecraft Attitude Determination and Control,* edited by J. R. Wertz, Reidel, Dordrecht, The Netherlands, 1978, pp. 257–266.

[6]Junkins, J. R., White, C. C., and Turner, J. D., "Star Pattern-Recognition for Real-Time Attitude Determination," *Journal of the Astronautical Sciences,* Vol. 25, July–Sept. 1977, pp. 251–270.